ECE 344

MICROWAVE FUNDAMENTALS PART1-Lecture 6

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Examples on

- SWR for short circuit , open circuit Transmission line
- Slotted line and determination of unknown impedance
- Input impedance for Terminated TL with shunt element or equivalent stub.
- Coaxial, characteristic impedance and Attenuation.

Experiment 1

- To determine the Standing Wave-Ratio and Reflection Coefficient

Experiment 2

- To measure an unknown Impedance with Smith chart

Experiment 3

- To determine the frequency & wavelength in a rectangular Waveguide working on TE10 mode

Previous lectures Equations

$$\gamma = [(R + j\omega L)(G + j\omega C)]^{1/2} = \alpha + j\beta$$

$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$\beta = \frac{2\pi}{\lambda_g}$$

$$v_p = \frac{\omega}{\beta}$$

$$Attenuation\ in\ dB\ \ for\ matched\ TL = 20\log e^{-\alpha l}$$

$$Zin = Z_o \frac{1+\Gamma_L e^{-2j\beta l}}{1-\Gamma_L e^{-2j\beta l}} = Z_o \frac{1+\Gamma_{in}}{1-\Gamma_{in}}, \quad Zin = Z_o \frac{Z_L + jZ_o \tan\beta l}{Z_o + jZ_L \tan\beta l}$$

$$\Gamma_{in} = \frac{Z_{in} - Z_o}{Z_{in} + Z_o}, \quad \Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}$$

$$short\ circuit\ load \quad Zin = jZ_o \tan\beta l, \quad open\ circuit\ load \quad Zin = -jZ_o \cot\beta l$$

$$\lambda/4\ transformer \quad Zin = Z_{ch}^2 / Z_L$$

$$V(z) = \left|V_o^+\right| e^{j\Phi^+} e^{-\alpha z} e^{-j\beta z} + \left|V_o^-\right| e^{j\Phi^-} e^{\alpha z} e^{j\beta z} \rightarrow lossless \quad V(z) = V_o^+ e^{-j\beta z}(1+\Gamma_L e^{2j\beta z})$$

$$\left|V(z)\right| = \left|V_o^+\right| \left|(1+\left|\Gamma_L\right| e^{j(\varphi - 2\beta l)})\right| \rightarrow \left|V_{\max}(z)\right| = \left|V_o^+\right|(1+\left|\Gamma_L\right|), \quad \left|V_{\min}(z)\right| = \left|V_o^+\right|(1-\left|\Gamma_L\right|)$$

$$\max\ at\ \varphi - 2\beta l_{\max} = 0, \quad \min\ at\ \varphi - 2\beta l_{\min} = \pi$$

Standing Waves -Matched

Matched Line ($Z_L = Z_o$), we had

$$Z_{in} = Z_o, \quad \Gamma_L = 0, \quad s = 1$$

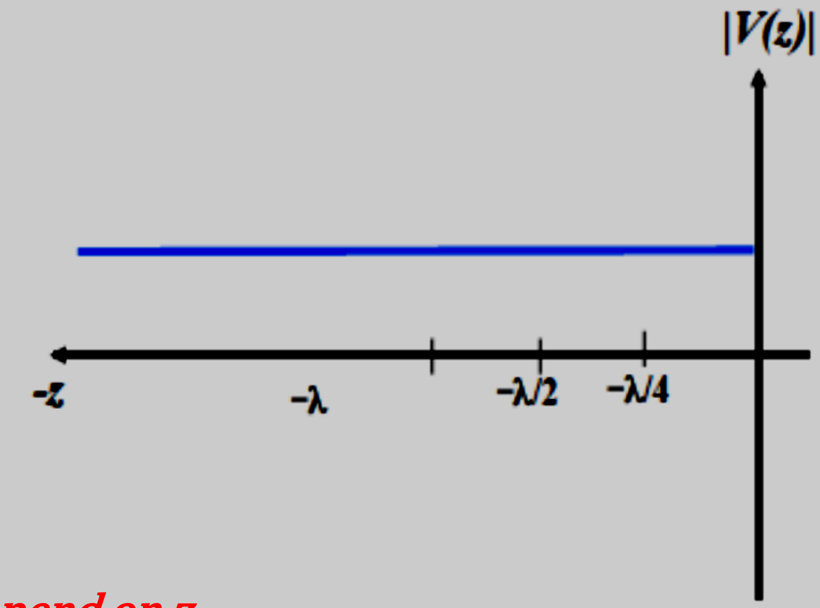
- So substituting in $V(z)$

$$V(z) = V^+[e^{j\beta l} + (0)e^{-j\beta l}]$$

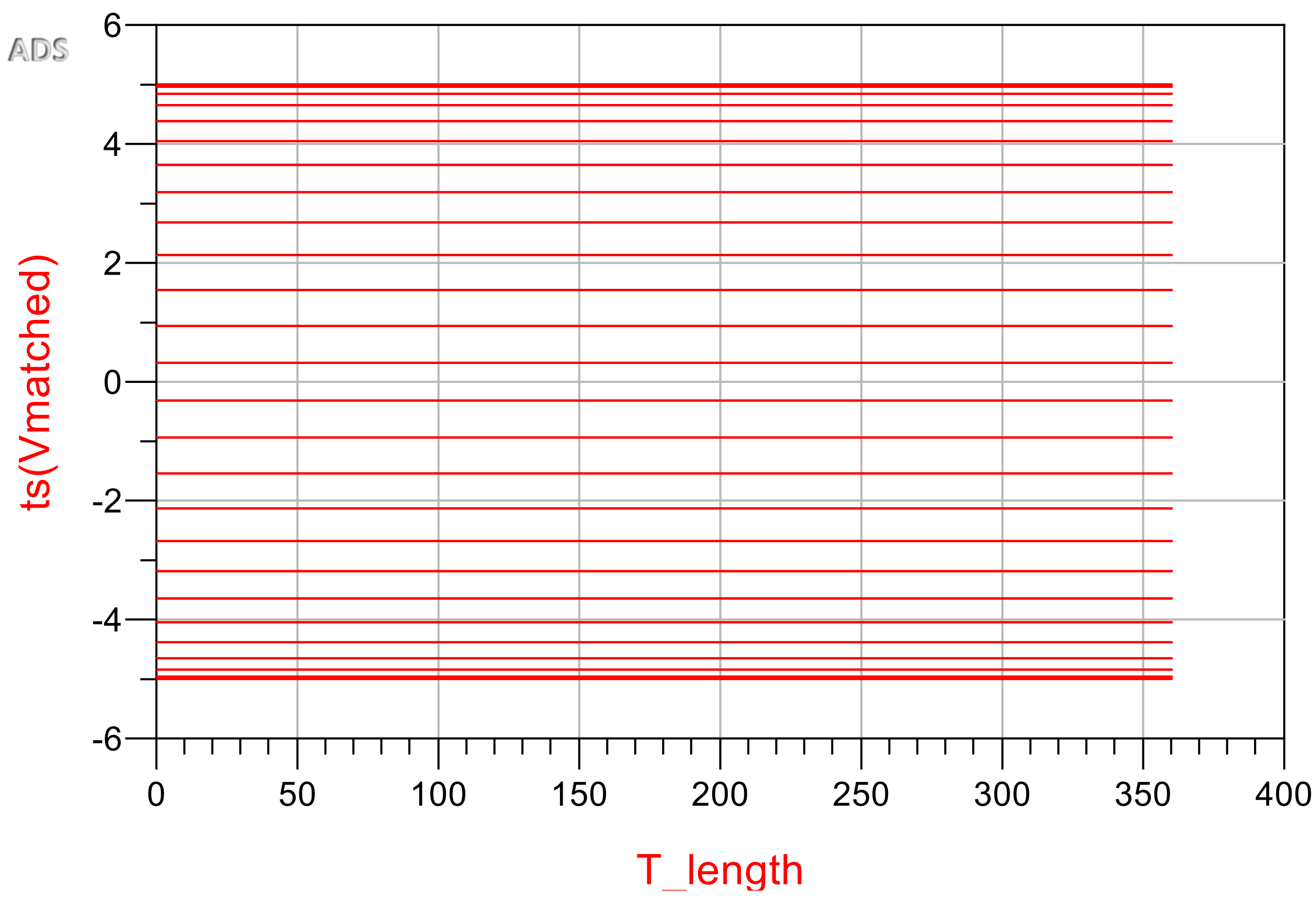
$$V(z) = V^+ e^{j\beta l}$$

$$|V(z)| = |V^+||e^{j\beta l}|$$

$$|V(z)| = |V^+|$$



$|V(Z)|$ *does not depend on z*



ADS
6
4
2
0
-2
-4
-6
ts(Vmatched)
0
50
100
150
200
250
300
350
400
T_length

Standing Waves -Short

Shorted Line ($Z_L=0$), we had

$$Z_{in} = jZ_o \tan\beta l\,, \quad \Gamma_L = -1, \quad s = \infty$$

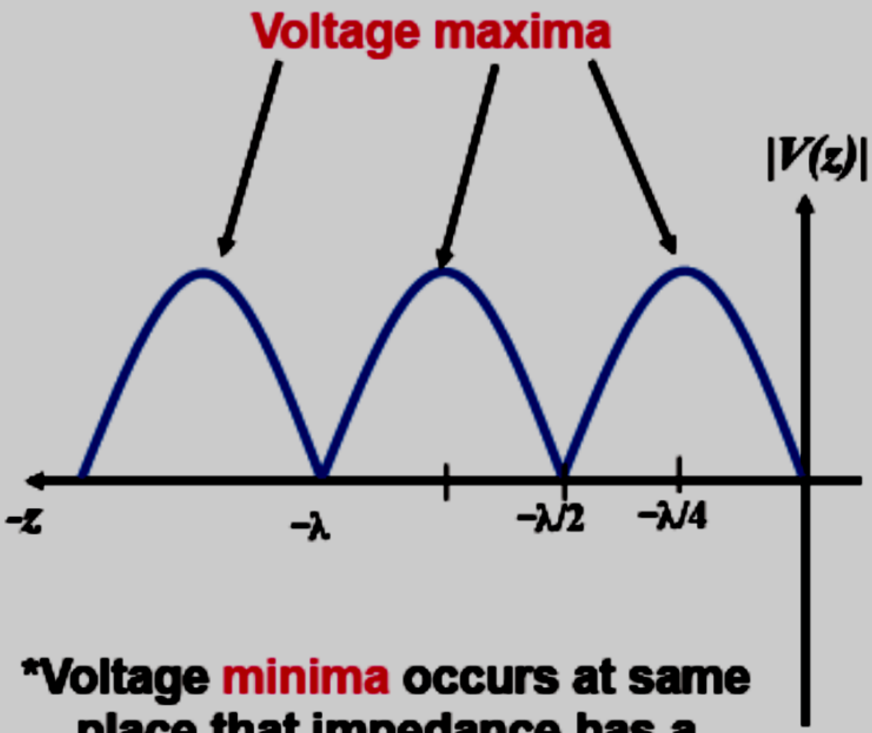
- So substituting in $V(z)$

$$V(z) = V^+[e^{j\beta l} + (-1)e^{-j\beta l}]$$

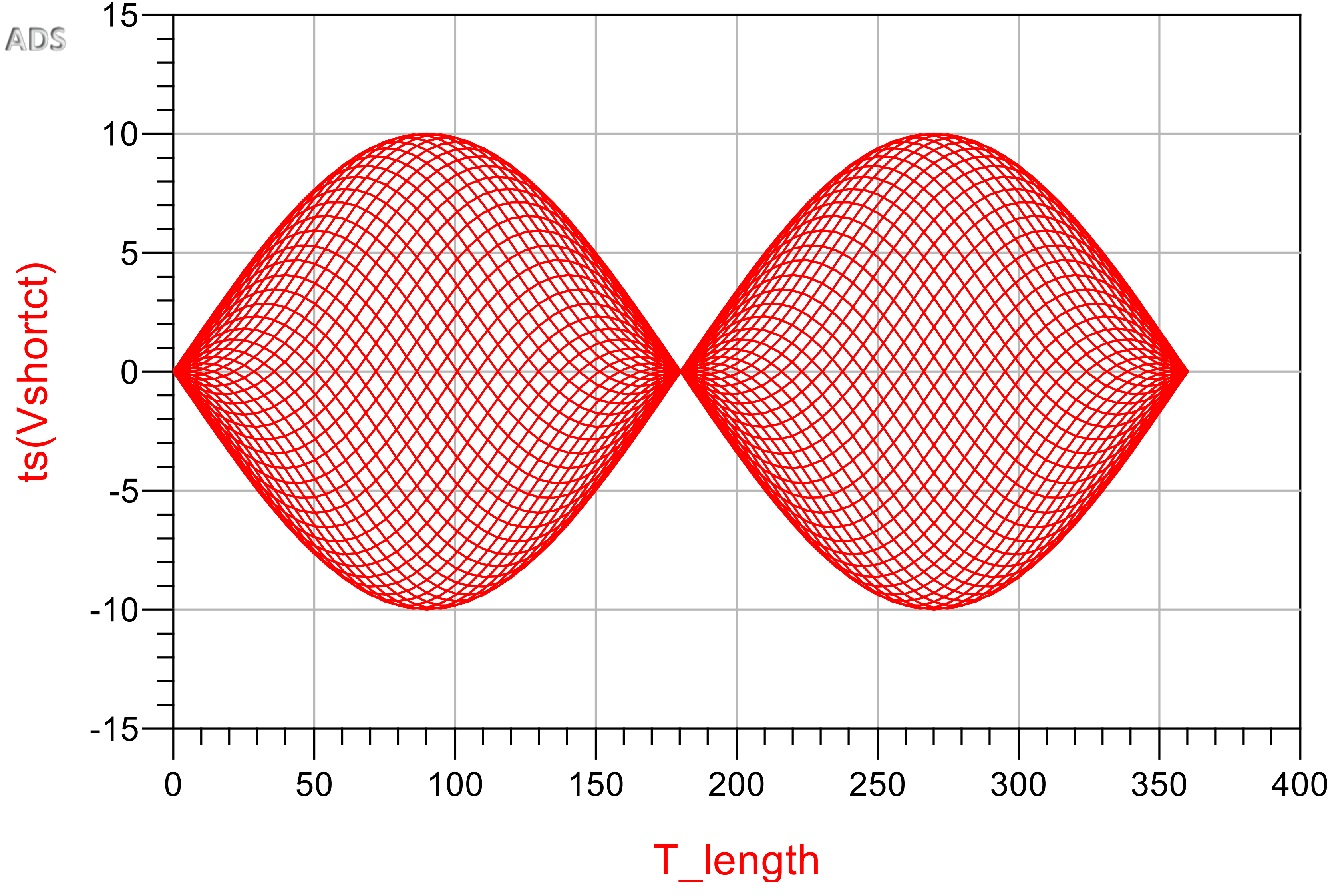
$$V(z) = V^+(2j\sin\beta l)$$

$$|V(z)| = |V^+||2\sin(\beta l)|$$

$$|V(z)| = |V^+|\left|2\sin\left(\frac{2\pi}{\lambda}l\right)\right|$$



*Voltage minima occurs at same place that impedance has a minimum on the line



ADS
15
10
5
0
-5
-10
-15
ts(Vshortct)
0
50
100
150
200
250
300
350
400
T_length

Standing Waves -Open

Open Line ($Z_L = \infty$) ,we had

$$Z_{in} = -jZ_o \cot \beta l\,, \quad \Gamma_L = +1, \quad s = \infty$$

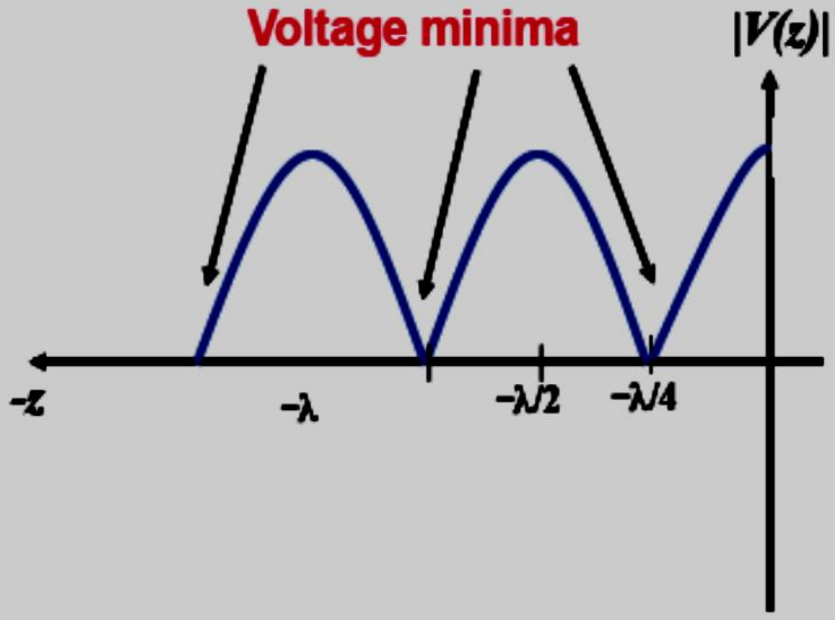
- So substituting in $V(z)$

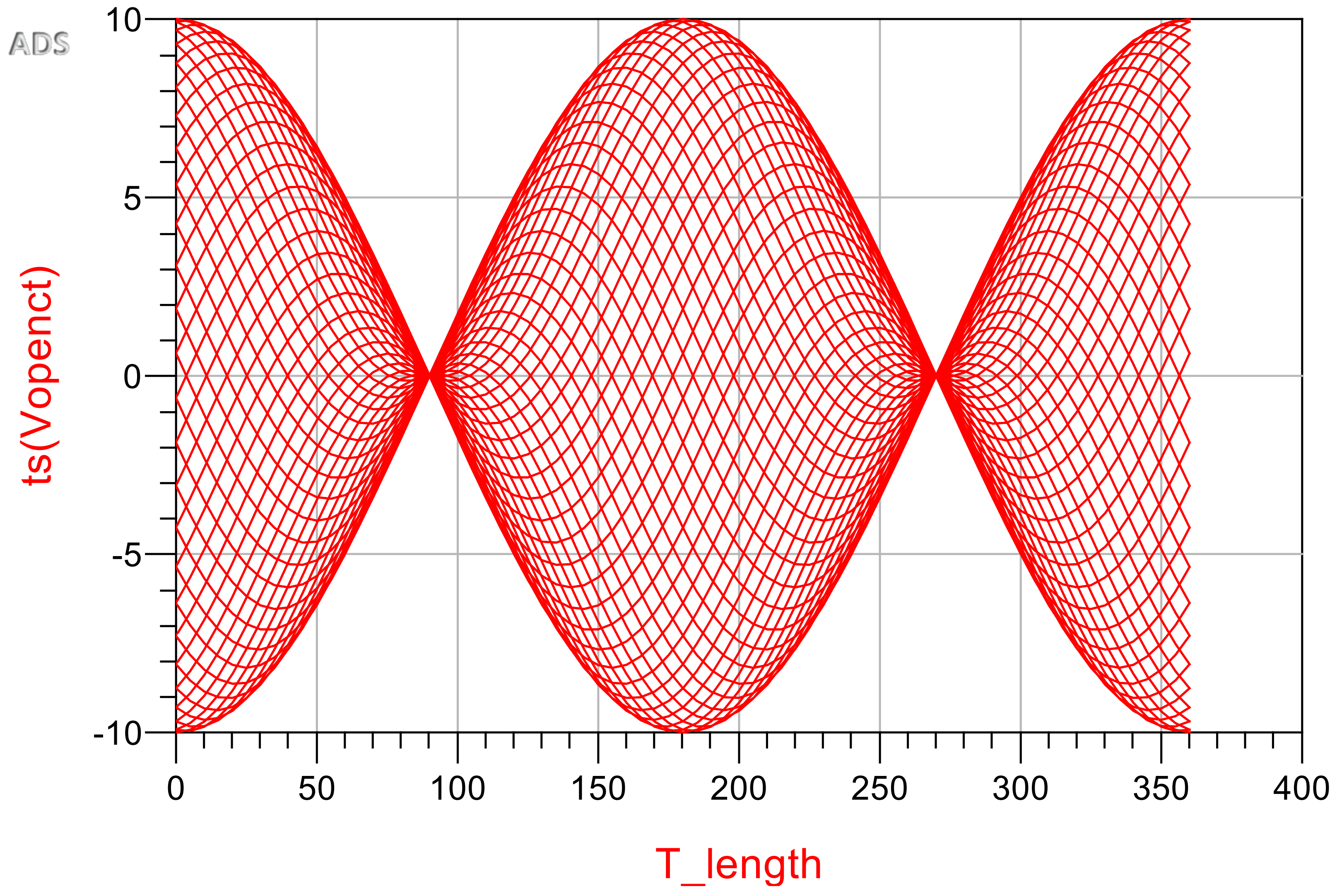
$$V(z) = V^+[e^{j\beta l} + (+1)e^{-j\beta l}]$$

$$V(z) = V^+(2\cos \beta l)$$

$$|V(z)| = |V^+||2\cos(\beta l)|$$

$$|V(z)| = |V^+|\left|2\cos\left(\frac{2\pi}{\lambda} l\right)\right|$$




ADS
10
5
0
-5
-10
ts(Vopenct)
0
50
100
150
200
250
300
350
400
T_length

VSWR

$$|V(z)| = |V_0^+| \left|(1 + |\Gamma_L| e^{j(2\beta z + \phi^- - \phi^+)})\right|$$

$\theta = \phi^- - \phi^+$ is angle of reflection coeff. at load

$$\max |V(z)| \text{ occured at } e^{j(2\beta z + \phi^- - \phi^+)} = 1 \; (or\ 2\beta z + \phi^- - \phi^+ = 2m\pi \quad \text{where m} = 0,1,2)$$

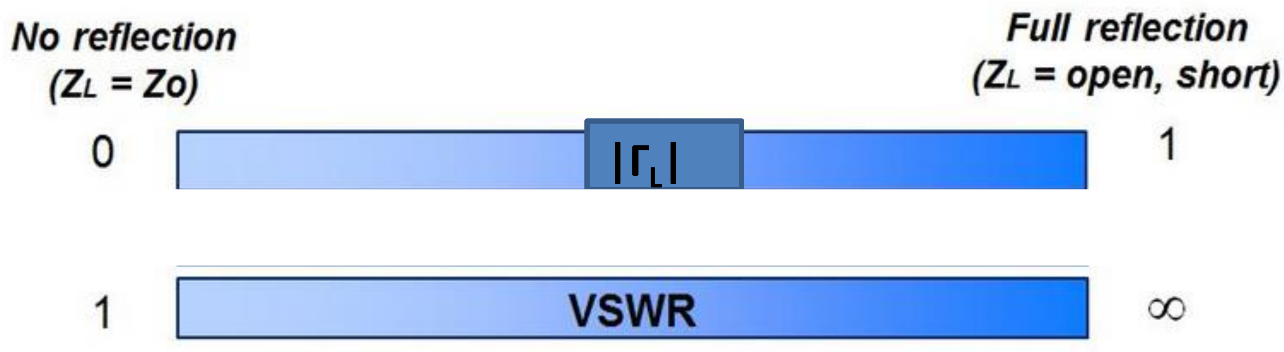
$$|V(z)|_{\max} = |V_0^+| (1 + |\Gamma_L|)$$

$$\min |V(z)| \text{ occured at } e^{j(2\beta z + \phi^- - \phi^+)} = -1 \; (or\ 2\beta z + \phi^- - \phi^+ = (2n+1)\pi \text{ where n} = 0,1,2)$$

$$|V(z)|_{\min} = |V_0^+| (1 - |\Gamma_L|)$$

$$\text{VSWR} = \frac{|V(z)|_{\max}}{|V(z)|_{\min}} = \frac{(1 + |\Gamma_L|)}{(1 - |\Gamma_L|)}$$

VSWR depend on mag of gamma only

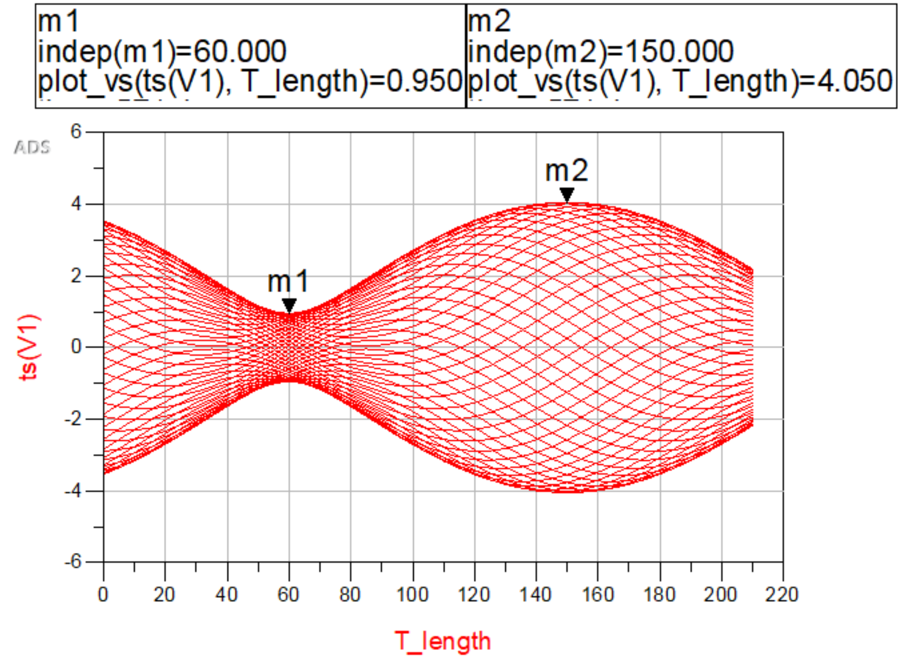


Example:

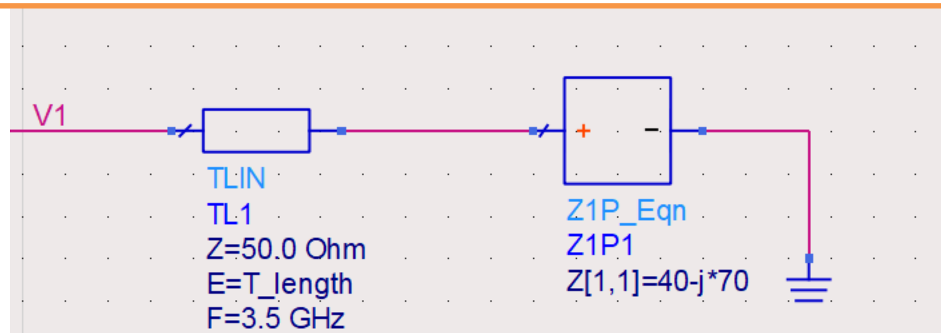
The results of a slotted-line experiment are plotted in the following figure. The max at length of the line = 3.57 cm ; And the min at length 1.43cm.

its characteristic impedance is 50 Ω. Find

- -frequency of signal on the line
- -the reflection coefficient at load
- -the load impedance
- -input impedance
- -input reflection coefficient



solution



freq	Γ_{load}	Γ_{in}	Zin
3.500 GHz	0.620 / -60.255	0.620 / -120.255	30.742 / -60.128

Solution: $\ell_{max} - \ell_{min} = \lambda/4 \rightarrow (3.57 - 1.43)\,cm \times 4 = \lambda \rightarrow f = \dfrac{0.3G}{0.0856} = 3.5\ GHz$

$\lambda = 8.56\,cm$

- From Fig $\dfrac{V_{max}}{V_{min}} = \dfrac{1+|\Gamma|}{1-|\Gamma|} \rightarrow \dfrac{4.05}{0.55} = \dfrac{1+|\Gamma|}{1-|\Gamma|} \rightarrow |\Gamma| = 0.62$

- $\Phi_L - 2\beta \ell_{min} = \pi \rightarrow \Phi_L = \pi + \dfrac{2\times 60}{180}\pi = \dfrac{5\pi}{3} = 300^\circ$

$\Gamma_L = 0.62\angle 300 \rightarrow Z_L = 50 \times \dfrac{1 + 0.62\angle 300}{1 - 0.62\angle 300} = 40.3 - j70.2$

From Fig:

- $\beta \ell = 210^\circ \quad \therefore \Gamma_{in} = \Gamma_L e^{-2j\beta \ell} = 0.62\angle 300 - 2\times 210 = 0.62\angle -120$

- $Z_{in} = Z_0 \dfrac{1+\Gamma_{in}}{1-\Gamma_{in}} = 50 \dfrac{1 + 0.62\angle -120}{1 - 0.62\angle -120} = 15.4 - j26.79 = 30.87\angle -60.17$

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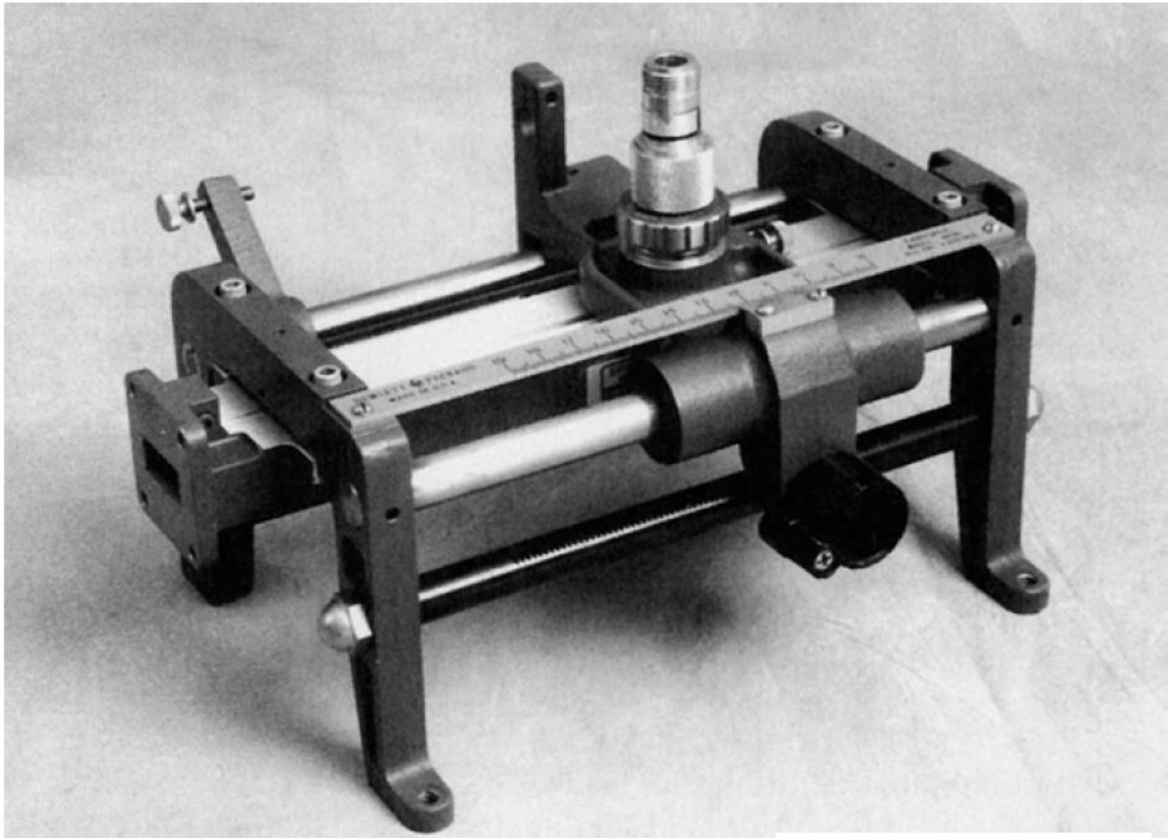
The slotted line

slotted line is a transmission line configuration that allows sampling of the electric field amplitude of a standing wave on it when it is terminated by a load.
Measuring SWR and the distance of the first voltage minima from the load allows us to compute the unknown load.



waveguide slotted line.

Using Slotted Line to Measure an Unknown Impedance:

1- From the SWR meter get :

$$|\Gamma_L| = \frac{SWR-1}{SWR+1}$$

2- Measure successive minima and maxima to get the frequency

$$l_1 - l_2 = \lambda / 4 \rightarrow \beta = \frac{2\pi}{\lambda}$$

3- Use l_{min} to get phase of reflection coefficient

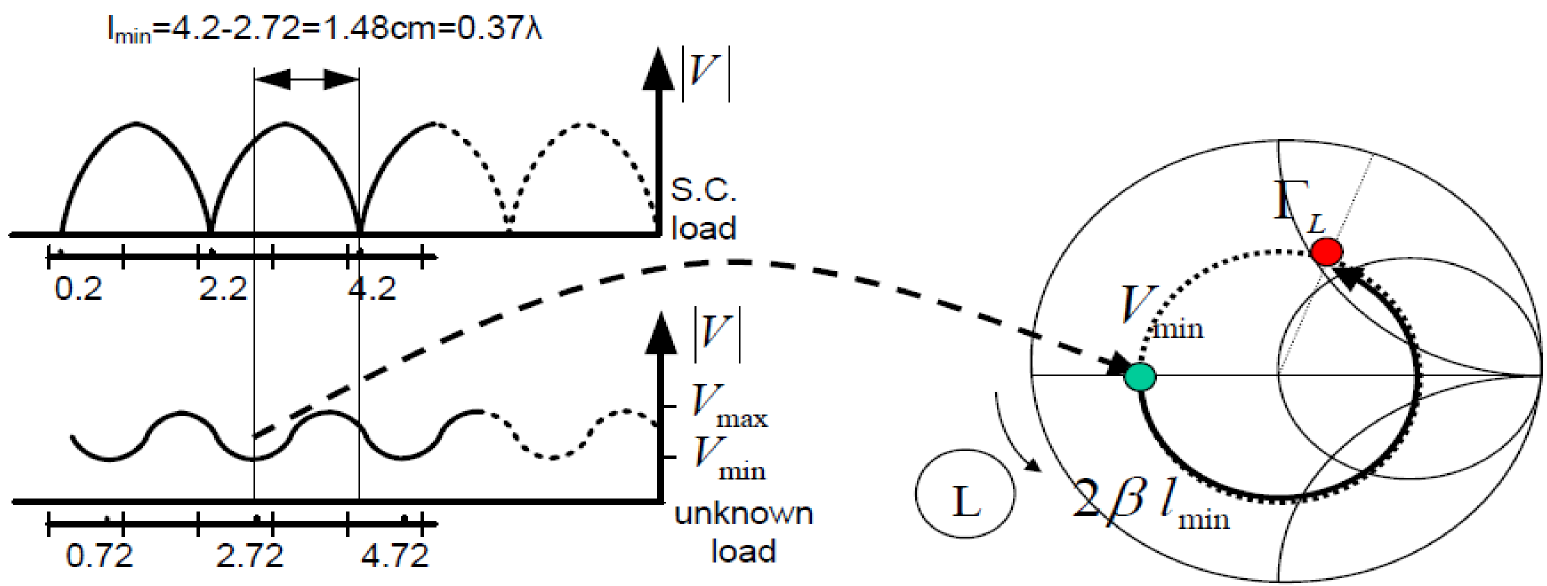
$$\varphi - 2\beta\, l_{min} = \pi \rightarrow \varphi$$

4- Compute Z_L or use smith chart to get Z_L

$$\Gamma = |\Gamma| e^{j\varphi}$$

$$Z_L = Z_0 \frac{1+\Gamma}{1-\Gamma}$$

Ex. VSWR=1.5, find Γ_L



$$\frac{\lambda}{2} = 2cm \rightarrow \lambda = 4cm$$

$$VSWR = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} \rightarrow |\Gamma_L| = \frac{VSWR-1}{VWSR+1} = \frac{1.5-1}{1.5+1} = 0.2$$

$$\angle\Gamma_L = 180^o + 2\beta l_{min} = 180^o + 2\frac{2\times 180^o}{4}\times 1.48 - 360^o = 86.4^o$$

$$\Gamma = 0.2e^{j86.4^\circ} = 0.0126 + j0.1996.$$

The load impedance is then

$$Z_L = 50\left(\frac{1+\Gamma}{1-\Gamma}\right) = 47.3 + j19.7\Omega.$$